Sunscapes: ‘solar envelopes’ and the analysis of urban DEMs
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\textbf{ABSTRACT:} The aim of this paper is twofold: first, it simplifies ‘solar envelope’ calculations, making them easier to carry out over extensive urban areas; second, the proposed routines refine the definition of solar envelopes so that they reflect more accurately the actual irradiation of the city and make results directly applicable in urban planning. Both targets can be achieved if we use a new technique for urban analysis, based on the processing of urban Digital Elevation Models (DEMs). The second question, in particular, can be addressed by introducing the concept of ‘iso-solar surfaces’ – surfaces that extend the concept of solar envelope through energy considerations. Our early experiments suggest that iso-solar surfaces may be an effective way to quantify urban irradiation and illumination. As an example, results are presented for the Trade Fair site in Milan, Italy, one of the largest real estate developments currently being implemented in Europe.
1. Introduction

“The sky is dominant, foremost among all things, this same sky which is the source of climate. The angle at which the sun meets the meridian imposes basic limitations on man’s behaviour. […] I believe that man’s yearning for light is natural. In a temperate climate I would not balk at having that light, even the sun itself, flooding the home.” (Le Corbusier, 1961)

Access to sunlight and daylight has always been one of the key generators of urban form. This is evident in vernacular architecture (Rapoport, 1969) and has been the source of inspiration for many architects in recent times. Furthermore, access to sunlight has become a matter for stringent urban regulations in modern times: bylaws, such as those implemented in the planning of New York and other dense cities throughout the world, aim to guarantee access to sunlight and daylight through the concept of ‘obstruction angles’. Obstruction angle is defined as the smallest angle with the horizontal under which the sky can be seen from the lower edge of a vantage point, usually an opening in a building. More sophisticated methodologies aiming to quantify direct sunlight collected by buildings precisely, were not used in the past because of computational complexity.

It was Knowles (1974) who refined the obstruction angle rule and introduced the well-known concept of the ‘solar envelope’: a 3-dimensional surface, on a given site, that does not obstruct more than $n$ hours of sun onto adjacent sites (see more detailed definition below). The idea was later extended by Capeluto and Shaviv (1997), who distinguished between ‘solar rights envelopes’ and ‘solar collection envelopes’ – the
former resembling Knowles’ initial definition, and the latter examining the total number of sun-hours collected (as opposed to cast) by a particular urban 3-dimensional surface.

While attractive from a theoretical standpoint, solar rights and collection envelopes have not yet been widely used in urban planning, as they pose a number of problems. First, they are difficult to calculate over extensive urban areas. Even recent software that couples solar envelope calculations with traditional CAD software (see for instance Noble and Kensek, 1998) or other simplified approaches (for instance Pereira et al., 2001), fails at the city scale because of its complexity. Solar envelopes seem useful for understanding the potential of regular urban grids, such as those found in many American cities (Knowles, 2003); but when used on large and irregular urban sites, such as the case study presented in this paper, they become excessively costly from a computational point of view.

Second, solar envelopes at present do not take into account energy considerations. They are defined in terms of discrete numbers of hours of sun or shadow, with little reference to actual radiation or illumination levels. However, an hour of sun at midday has different effects from an hour of sun at dawn. The idea of energy-weighting solar envelopes was hinted at in seminal work by Knowles (1974, 1981), but later abandoned due to computational difficulties. In fact, Knowles suggests weighting incident rays of light considering varying sun altitudes during the day, in order to approximately quantify the amount of energy that gets through the atmosphere (1981, p.57).

This paper investigates whether both limitations mentioned above could be overcome using new techniques for urban analysis, based on the image processing of very simple raster models of urban form: the so-called ‘digital elevation models’ (DEM). Ratti and
Richens (2004) have shown how DEMs are very conducive to the environmental analysis of extensive urban areas, while the authors of this paper (Ratti and Morello, 2005) have described how they could contribute to the assessment of the radiative landscape in cities. The question is if these techniques could contribute to solar envelope calculations, making them more accessible for urban areas and, moreover, if they could allow weighting of solar envelopes based on the actual irradiation and illumination of the city.

The answers to the above questions are discussed below through the introduction of the concept of ‘iso-solar surfaces’: 3-dimensional geometric envelopes which receive equal amounts of solar energy. Two types of iso-solar surfaces can be defined: iso-solar rights surfaces (ISRS) and iso-solar collection surfaces (ISCS). The first indicate at each point in space the maximum allowable height of buildable volumes in order to guarantee solar irradiation on adjacent sites; the second represent at each point the minimum elevation from the ground for collecting a given amount of solar radiation. Both are calculated and compared on a case study in Milan, Italy: the Trade Fair site, which is currently one of the largest real-estate developments in Europe, following an international competition which was won by architects Zaha Hadid, Arata Isozaki, Daniel Libeskind and Pier Paolo Maggiora.

A sketch digital model of the winning design scheme and a DEM of the case-study site are shown in Figure 1. The area considered is a square 1000 meters wide. On the 255,000 square meters designated for redevelopment, the winning design scheme proposes the construction of three towers around a central park. The plot is completed by lower residential buildings facing the existing urban fabric at the boundaries of the Trade Fair zone.
The case study is interesting, as it allows us to compare five very different design schemes presented for the competition as well as the winning scheme.

![Figure 1](image1.png)

**Figure 1** – Left: A digital sketch model of the winning scheme for the Milan Trade Fair redevelopment case study site, designed by architects Zaha Hadid, Arata Isozaki, Daniel Libeskind and Pier Paolo Maggiora; the redevelopment area with new buildings is highlighted in gray. Right: the DEM of the design project represented in the axonometric view; the gray color is proportional to the height of the buildings as shown in the colorbar.

## 2. Context: solar laws in the praxis of urban planning and in research

Traditional solar laws are based on simple angular criteria, such as obstruction angle rules (see for instance local bylaws). The case of New York’s Zoning Law of 1916 is emblematic: it was enacted as a reaction against the negative environmental effects caused by excessive building height and density in central Manhattan. It stated that construction can “proceed up to a certain height; then the building must step back from the plotline at a certain angle to admit light to the streets. A tower may then carry 25 percent of the plot area to unlimited heights” (Koolhaas, 1978). As a result, a maximum volume resulting from legal prescriptions could be built on each site. This is well represented in the renderings of Hugh Ferriss, which describe a city of maximal
envelopes, “a ghost town of the future […] a collection of 2028 colossal phantom ‘houses’ that together form a Mega-Village” (Koolhaas, 1978). A similar law based on an obstruction angle of 60 degrees applies to the city of Milan. For instance, if it were to be applied to the Trade Fair case-study site examined in this paper, it would give a maximum buildable volume such as that shown in Figure 2, on the right. The computed phantom Milan trade fair site represents the maximum buildable volume that can reach a height of over 400 meters\(^1\). In the absence of an internal road system, the entire plot does not necessarily require a further subdivision and the 60 degrees obstruction rule can be applied as if a single building would occupy the entire area. Of course, the single phantom building constitutes the extreme example for speculation. All five design schemes suggest the construction of numerous buildings, each of which would require the verification of local bylaws.

These bylaws represent an easily applicable measure to ensure a minimal environmental quality. However, one of the limitations is that they only consider a rudimentary estimate of the openness of the city to the sky, without taking into account real sun paths. As a result, there have been a number of attempts to better incorporate solar access into urban planning, such as the first solar laws enacted at the end of the Seventies in California (Erley and Jaffe 1979; Thayer, 1981).

\[^1\] The algorithm used for the computation of the maximum buildable volume on the case-study area according to local bylaws was based on the shadow-casting script introduced later.
Figure 2 - The maximum buildable volume on the Milan Trade Fair case-study site based on a 60 degree obstruction rule.

The most notable theoretical effort is the one that led to the concept of ‘solar envelope’, which made its appearance during the energy crisis of the 1970’s. Initially developed by Knowles, the solar envelope on a given site is defined as “the volumetric limits of building that will not shadow surroundings at specified times” (Knowles, 1981). In other words, given a certain site in an urban context, the solar envelope defines the maximum built height that can be reached on that site without compromising the neighbouring buildings’ solar accessibility. The latter is defined as the minimum number of hours of sun, during critical periods of the day and the year. More recently, the concept of solar envelope was extended by Capeluto and Shaviv (1997), who distinguished between solar rights envelope (SRE) and solar collection envelope (SCE). The SRE defines the maximum height of a building in order not to violate the solar rights of any of the
neighbouring buildings during a given period of the year; it is basically equivalent to Knowles’ definition and refer to the pioneer solar rights laws first introduced in California during the Seventies (Erley and Jaffe 1979; Thayer, 1981) that forced new construction to respect the solar accessibility of neighbouring buildings by prohibiting the obstruction of the skyspace, i.e. a specific solid angle above solar panels installed on roofs. The SCE defines the lowest possible surface to locate windows and solar collectors so that they are not obstructed by neighbouring buildings, during a given period of the year. This is, in a certain sense, a symmetrical parameter to the SRE: it describes the overshadowing of a neighbourhood on a given site. The SRE and SCE can be interpreted as the upper and lower boundary of a ‘solar volume’, which represents the portion of space where new developments could be allowed without reducing the solar access of the neighbouring buildings while guaranteeing sufficient solar access to the development. While plausible from a theoretical standpoint, they are difficult to compute on large urban sites, because the complex geometry of the urban fabric requires a fine level of detail thus generating a time consuming computation.

3. Calculating solar envelopes on a DEM

The image processing of urban DEMs could provide an alternative and efficient way to calculate solar envelopes over complex urban geometry as proposed in the introduction to this work. The subroutine presented below is derived from the simple shadow-casting script introduced by Ratti and Richens (2004) and represents the basis for further implementations of macros dealing with the translation of DEMs. This procedure is
shown in Figure 3. First, the three components of the vector pointing towards the sun are defined. Then, we compute the components of an opposite vector, scaled so that the larger of the x and y components is just 1 pixel, and the z component is adjusted to the image calibration (Figure 3a). If we translate the DEM (Figure 3b) by the x and y components, and simultaneously reduce its height by subtracting the z component, we get part of the shadow volume. If we continue translating and lowering by multiples of this vector, and take the maximum of this volume with that previously calculated, we build up the whole shadow volume (Figure 3c). The process can be stopped when all levels are zero or the translation has shifted the volume right off the image. To reduce the shadow volume to an actual map of shadows on the roofs and ground level of the city, the original DEM is subtracted from the shadow volume (Figure 3d). Pixels with negative or zero values are in light; positive values are in shade.

Figure 3 - The computation of the shadow volume through the image-processing of DEMs; from left: (a) the translation macro, (b) the DEM for a square of height \( z \), (c) its shadow volume where gray levels indicate the heights of each pixel and (d) the map of shadows.

The above described routine, which is presented in the Appendix of this paper, allows the rapid computation of the so-called ‘shadow volume’, i.e. the volume of air that is in
shadow over a given urban DEM. The interest of this approach is that the raster analysis is impressively fast and can deal with great vectorial complexity (due to the fact that processes are reduced to simple operations on pixels). Ratti and Richens’ algorithm computes shadows for an arbitrary lighting angle. The next stage is to add a procedure to calculate shadows from the sun for any given latitude, time of year, and time of day, using the standard astronomical formulae (Szokolay, 2004). This allows simulations for an entire day or a number of days and thus enables the calculation of the following parameters:

- **Solar envelope.** According to Knowles’ definition, the calculation is based on the intersection of four boundary shading conditions that generate a pyramid-like volume. The north face of the volume is defined by shadowing at noon during the winter solstice, the south face by shadowing at noon during the summer solstice. The west and east boundaries are chosen from daily values, depending on the number of hours of sun that need to be guaranteed on the surroundings and that can be chosen from different seasonal solar exposures (i.e. winter solstice or spring equinox). The four boundary conditions are then calculated on a DEM. First, Ratti and Richens’ (2004) shadowing script has been modified, by reversing the sign of the shadowing increments and of the image shifts: this process would be equivalent to casting fictitious shadows that start at the base of existing buildings and move upwards in the direction of the sun. The conceptual scheme of the script is shown in Figure 4. The subroutine is then applied to all boundary conditions. As a result, four matrices, one for each cut-off time defined for Knowles’ SE, are obtained. The solar envelope is finally produced by choosing out of

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2 The number of hours of direct sun to be guaranteed on the urban facades depends mainly on the environmental and irradiation standards that can be defined by local bylaws, according to the functional use of the building and its geographical location. For example, a performance standard can establish at least three hours of direct irradiation in winter on living rooms in mid-latitude locations.
these latter four matrices the minimum values (i.e. the most restrictive condition) on a pixel-by-pixel basis. For example, referring to Figure 4, on each point of the site area, the lowest value among all the projections of shadows intersecting at that point (on the represented section of Figure 4, point 2 constitutes the most restrictive condition on the left boundary of the site area.). In this way the characteristic pyramidal shape of the solar envelope emerges.

Figure 4 – The conceptual construction of the script for calculating solar envelope through the image processing of DEMs. The shadowing function is applied shifting the DEM starting at the base of existing buildings towards significant cut off sun angles.

- **Solar rights envelope.** A similar method to the one described above generates the solar rights envelope. In this case, different images are created for each hour of the day, whereby intensity values of pixels correspond to the height of the shadow volume in meters. Out of these hourly images the minimum values on a pixel by pixel basis are stored in a separate array, thus defining the surface that never casts shadows on the surroundings. Depending on how many hours of radiation per day are taken into account in the computation, we can establish variable solar surfaces: for instance, if we decrease
the number of hours of direct irradiation we want to guarantee on the surrounding sites, we will increase the height of the SREs.

- Solar collection envelope. This parameter is in a certain sense easier to calculate, as Ratti and Richens' (2004) script does not need to be modified and can be used in its normal shadowing form. In order to obtain the solar collection envelope, shadows are cast in a standard way from the top of existing buildings. A shadow volume is calculated for every hour of sun in the day. The results, stored in a final matrix, are sorted out by increasing values on a pixel-by-pixel basis. The highest value defines the optimum condition of irradiation.

Representations of the solar envelope, the solar rights envelope and the solar collection envelope, calculated on the Milan Trade Fair case-study site, are shown both in top and axonometric views in Figure 5. As anticipated, the first two have a pyramid-like shape, with the solar rights envelope being smoother and lower as it represents a more restrictive condition than the solar envelope (greater number of hours on which it is calculated). Solar rights envelopes are characterized by higher profiles toward the south orientation and decreasing, sloped surfaces facing north. In particular, top views reveal the influence of the built fabric in determining the shape of the solar envelopes. In fact, all envelopes are higher in correspondence to open spaces and streets and the orientation of the urban street network clearly emerges. The solar collection envelope has a concave shape and decreases when we move toward the centre of the empty site, since there the same solar irradiation can be reached at lower heights. The space defined by the solar rights (upper surface) and the solar collection envelope (lower surface) corresponds to the solar volume.
The solar rights envelope obtained in Figure 5b can be re-processed with the shadow-casting script in order to visualize how the shadows generated by this pyramid-like volume stop exactly at the base of existing buildings, thus not affecting the neighborhood. The map of shadows for the solar rights envelope calculated on the 21st December is reported in Figure 6. An application of the use of these parameters in planning is shown in Figure 7, where the winning masterplan for the site has been superimposed onto the solar rights envelope, showing the amount of volume which would infringe solar rights on the neighboring sites (most of the upper part of the three planned skyscrapers).
Figure 5 - From top to bottom: Top and axonometric views of (a) Knowles’ solar envelope, (b) the solar rights envelope and (c) the solar collection envelope, calculated on the Milan Trade Fair case-study site.
Figure 6 - The verification of the SRE reported in figure 5(b) through the shadow casting on the solar volume; as seen on the top view on the right where the footprints of buildings are visualized on the plan, shadows do not affect the neighborhood on the 21st December (7 hours of daylight) and stop at the base of existing buildings.

Figure 7 - The superimposition of the winning scheme for the Milan Trade Fair redevelopment project and the solar rights envelope reported in figure 5(b) which shows the amount of volume that would infringe solar rights on the neighbouring sites: for instance, the upper part of the three skyscrapers exceeds the solar envelope boundaries.
4. Extending solar envelopes: the concept of ‘iso-solar surfaces’

In their standard form, as calculated above, solar envelopes are quite flexible (Littlefair, 1998), but have a limitation: they do not take into account the angle of incidence of sunlight and/or its intensity. In other words, they are created by simply adding together binary images, 0 or 1, sun or shade. An hour of sun at midday and one at dawn count as the same, although they are very different from an energy point of view.

As mentioned above, Knowles (1981) suggested weighting different amounts of energy received at different times of the day/year by the sine of the sun’s altitude: “The functional time period for a solar envelope must be specified in response to a clear purpose. Time periods seem easier to specify for energy purposes than for quality of life. As society increasingly values solar access as a right, however, we may find that our purposes for solar access are more clearly qualitative. This discussion relates useful periods of solar access to energy, which requires definition of the term “useful” in relation to the technology of energy conversion and to the movement of the sun”. (Knowles, 1981).

In addition to the fact that it does not weight solar radiation based on incident angle, the solar envelope concept has another limitation: it does not take into account light diffused by the sky-vault\(^3\), which plays an important role in urban areas and could radically change results. As a result, this paper proposes to move the focus from the concept of ‘sunlight’, used for the calculation of solar envelopes, to the concept of ‘skylight’, which enables the quantification of energy coming from the unobstructed sky vault. The
question is how such an approach can be quantified. The intention to add some energy-oriented considerations on solar envelopes led us to the concept of what we have called iso-solar surfaces: 3-dimensional geometric envelopes representing the same amount of received solar energy. As in the case of solar envelopes, it is proposed here that two types of iso-solar surfaces are distinguished: iso-solar rights surfaces and iso-solar collection surfaces:

- Iso-solar rights surfaces are defined as 3-dimensional geometric envelopes which describe the maximum height for buildable volumes to preserve a given amount of irradiation on adjacent sites. Increasing values of radiances can be chosen, each of which will correspond to decreasing buildable heights.

- Iso-solar collection surfaces define the lowest possible surface that will collect a given amount of solar radiation. This parameter is complementary to the latter and describes the effect of shadowing on a given site by neighbouring buildings.

5. Calculating iso-solar surfaces

In this section the calculation of iso-solar surfaces is presented. The first decision to make is the choice of the sky model. In the case of this paper, sky data was provided by Raphael Compagnon (Figure 8, Table 1), whose previous work led the way in radiation and illumination mapping of statistical sky (Compagnon, 2004). In particular, the software Meteonorm, which provides the daylight mean radiances values [W/m²] of a particular sky region, was used. As a result, it is possible to compute the annual (or

\[ alert: The sky vault is a representation of the sky as a semi sphere located above a vantage point. Using this sky model, it is possible to subdivide the vault into a finite number of subareas and assign to each of them a value of solar radiances. \]
monthly) mean horizontal irradiance [W/m²] and the annual horizontal irradiation
[kWh/m²], calculated over the number of daytime hours determined for any given specific
region. The sky is discretised subdividing the hemisphere model into 145 patches of
given altitude, azimuth and irradiation value, as shown in Figure 9. The concept behind
this model is based on a previous scanning pattern for sky photometry first introduced by
Tregenza (1987), whereby 151 resulting circular zones covering about 70% of the
hemisphere were defined.

Having defined the sky model, the calculation of iso-solar surfaces proceeds in a similar
way to that explained in Section 4 above. The subroutines used to compute the SRE and
the SCE are not applied to given sun positions, but to the 145 zones of the statistical sky
and weighted with each corresponding energy value. Instead of 0s and 1s, the
corresponding irradiation values are obtained.

The storage and processing of results, however, is different and requires some
additional thought. In this case matrices are not made of discrete 0 and 1 values, but a
3-dimensional scalar field containing irradiation values. The easiest way to cope with
them would be to store them as voxel spaces⁴. Then, iso-solar surfaces for different
irradiation values can be identified using standard segmentation algorithms applied to
voxels.

The dimensions of the voxels are defined based on the level of precision required. It
should be noted that the technique presented here is not dependent on DEM resolution.
As in every raster system, the same algorithms could be used at smaller pixel

⁴ A voxel, or volumetric pixel, is a volume element, representing a value in 3-dimensional space. This is analogous to a
pixel, which represents 2-dimensional data.
dimensions. The size we choose here is a compromise between speed of computation and presentation requirements. If the purposes of investigation require very detailed results for a specific site, a simulation on a DEM portion could be run. For the first tests on the Milan Trade Fair case-study site each voxel was chosen as 5 x 5 x 5 m, a dimension that resulted in files of manageable size over an area that is 1000 x 1000 meters wide. In fact, for the purposes of this investigation, i.e. the determination of larger iso-solar surfaces, this level of accuracy is reasonable. For higher resolution, the analysis could be conducted cropping the site into small portions and using a more detailed voxel grid, taking into account the scale of buildings and considering openings and rooflines. In this case, the voxel space contains a series of superimposed layers at five meter intervals, each of which stores the collected energy quantity for its specific position. Results are shown in Figure 10 and Figure 11.

Furthermore, possible inaccuracies in the elevation of the DEM will translate into inaccuracies of the iso-solar surface. However, this will not change its form, thus not requiring probabilistic treatment of the computation as in the case of viewsheds (Fisher, 1995).

Figure 10 shows iso-solar rights surfaces based on an irradiation value of 260 W/m² applied over all the surroundings. Figure 10a applies the calculation of ISRSs over all urban open spaces and in Figure 10b the calculation is restricted to the case-study area. The differences, when compared with traditional solar envelopes, are striking. For instance, the surface is much smoother. This is due to the fact that shadows are calculated over the whole sky vault and not just for discrete sun positions. Thus, results are more accurate and representative of reality, as radiation levels change smoothly in space and not by discrete leaps. Consequently, the novel approach to develop solar envelopes presented here may have a significant application and impact in urban
planning, because these solar surfaces allow to precisely define how and where to locate urban volumes on an energy-based evaluation. In fact, we can shape buildings according to the context and the orientation of facades, providing a substantial improvement in terms of solar collection compared to traditional bylaws; in other words, the proposed model permits to design more energy-efficient urban surfaces overcoming traditional, simplistic rules based on obstruction angles, whereby every orientation is considered in the same way.

The little surface wrinkles are due to small discretizations in the algorithm and in the sky model. Raising and lowering the chosen level of irradiation would generate different envelopes: the higher the solar access on the neighbourhood, the lower the height of the surface and vice versa.

Figure 11 shows iso-solar collection surfaces for values of 250, 270, 280, 282 W/m\(^2\) respectively. Again, one could note their smoothness when compared with solar collection envelopes. They look like a carpet laid on the top of the neighbouring buildings with the height increasing when the requirements on collected energy increase, reflecting in a fine tuned way the changes in the radiative landscape over a given site.

Iso-solar surfaces reveal themselves to be more useful for assessing the potential solar collection of urban surfaces than solar envelopes, because they visualize very clearly where suitable areas for installing solar or photovoltaic panels are located in the urban environment. For instance, if one were to use the iso-solar rights surfaces and iso-solar collection surfaces on the Milan case-study, the answer would be much more accurate than the cut off answer provided by solar envelopes in Figure 5. Different irradiation levels could be tested, but in terms of collected energy and cast shadows. Also, it would
be easy to perform the analysis using illumination instead of irradiation values, thus evaluating not only received energy but also the effect it has on the human eye\(^5\) simply by changing the values of the statistical sky.

Figure 8 - Stereographic view of the sky model (Compagnon, 2004); from left to right: diffuse component, direct component and global (i.e. diffuse + direct).

<table>
<thead>
<tr>
<th>Annual mean horizontal irradiance under the sky model for Milan [W/m(^2)]</th>
<th>284.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of daytime hours [h]</td>
<td>4357.0</td>
</tr>
<tr>
<td>Annual horizontal irradiation under this sky model [kWh/m(^2)]</td>
<td>1238.8</td>
</tr>
</tbody>
</table>

Table 1 - Solar data for Milan, Italy, computed using the software Meteonorm.

\(^5\) Illumination is the photometric equivalent of irradiation, as defined by the Commission Internationale de l’Eclairage (CIE).
Figure 9 - The subdivision of the sky hemisphere into 145 zones used for the calculation of the iso-solar surfaces (Compagnon, 2004).

Figure 10 - Iso-solar rights surfaces on the Milan Trade Fair case-study site based on an irradiation value of 260 W/m² applied to all the surroundings. Left: (a) calculation applied over all open spaces; right: (b) same calculation shown only inside the exact boundaries of the case-study site.
Figure 11 - Iso-solar collection surfaces calculated on a portion of the Milan case-study site; the chosen surfaces collect respectively (a) 250, (b) 270, (c) 280, (d) 282 \text{ W/m}^2. Their height increases with increasing irradiation levels.

6. Conclusions

This paper attempts to improve the solar envelope calculation in two ways: first, it introduces a new technique for making calculations of solar envelopes over complex urban sites easier and more precise; second, it explores energy-based surfaces with
reference to the actual irradiation/illumination of the city. In particular, a new method for calculating solar envelopes is introduced. It is based on the image processing of urban DEMs and is deployable in the context of extensive and complex case studies. For the first time it brings solar envelopes within easy reach of architects and planners, thus making them applicable to urban bylaws. It demonstrates the potential of DEMs for the analysis of the city, which has many advantages including computational affordability, flexibility, precision and the possibility of comparing results obtained from several algorithms.

Moreover, the concept of iso-solar surfaces has been introduced, defined and demonstrated by example. When compared to solar envelopes, iso-solar surfaces have a major advantage inasmuch as they allow a more accurate quantification of radiation or illumination values. Unlike solar envelopes, which are computed at arbitrary cut-off times, they enable the calculation of different irradiation levels and a more accurate assessment of the impact of built form on the accessibility to solar radiation.

Simulated solar gain through iso-solar surface indicators could become a new and more reliable way to promote performance standards in urban planning, replacing coarse obstruction angles. For instance, maps or 'sunscapes', could be defined and integrated within urban bylaws. Different performance indicators could be defined at different points, thus creating the basis for an integrated system to incorporate environmental considerations in cities.

Minor issues still remain, especially the question regarding the resolution of the pixel or voxel grid. The power of the calculation environment affects enormously the quality of outcomes. We are currently using Matlab and we are exploring ways to optimize the
computation process. Another point is the performance in the visualisation of results with the possibility to store results and export the matrices to other interfaces. Furthermore, user-friendly interfaces that do not require specific knowledge of programming could contribute in diffusing the proposed technique.

In this sense, the authors of the present article are planning to launch an initiative to disseminate algorithms for the analysis of urban DEMs and make them more accessible. First, the plan is to improve the user interface beyond Matlab’s command lines into a kind of GUI. Second, an open-source software initiative will be launched to make the algorithms part of the public realm and be used and modified by the interested community. As urban DEMs are becoming increasingly available at low cost – also thanks to recent initiatives such as Google Earth (2006) – the proliferation of analysis tools to run on them could have a great impact on design and planning.

We feel that this could lend new relevance to the vision outlined by Knowles in his seminal work: that of an adaptive architecture sculpted by environmental forces such as the rhythm of the sun. So far solar envelopes have been a possible way to do this. Iso-solar surfaces can probably improve them, moving the focus from a qualitative point of view to more quantifiable parameters (solar irradiation and illumination). As a result, a new aesthetic in architecture could be possible, based on natural forces and sun rhythms, in order to improve and rebalance the quality of life inside buildings and in the urban environment as well.
7. Acknowledgements

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Finally, the pertinent and precious comments by the three anonymous reviewers motivated us to improve the quality of this work and add clearness to the reader.
Appendix

Listing of the code in Matlab to cast shadows on a DEM.

% inputs: “a” is the DEM, see for instance Fig 3b
% expects also: scale, the vector “sun_positions” containing the coordinates of
% the position of the sun; “widthx”, “widthy” are the extension of “a” along the
% x and y axis;

% Initialization parameters: “dx”, “dy”, “dz” are the increments for each step;
% “temp” is a temporary matrix that collects the increments of the shadow volume
% on the DEM; “final” is the final matrix where the results are stored
dx = 0;
dy = 0;
dz = 0;
index = 1;
temp = zeros (sizey, sizex);
final = a;

% input solar data
altitude = sun_positions(hour,1);
azimuth = sun_positions(hour,2);

% start displaying the shadow casting process
h = imagesc (- final / (max (max (final))), 'erasemode', 'none');
colormap(gray);
truesize;

% loop showing incremental shadow casting
% method: offsets of built (black) pixels of “a” along the azimuth vector
% the loop goes on until the shadows’ increments “dz” touch the ground and the
% horizontal increments of the shadow “dx” and “dy” reach the limits of the image
while ((max (max (a))>=dz) & (dx <= widthx) & (dy <= widthy))

% incremental projections are calculated and stored in “temp”
dx = -round (index * cos (azimuth));
dy = round (index * sin (azimuth));
dz = index * tan (altitude) / scale;
temp(1 : widthx, 1 : widthy) = 0;

% we translate the DEM by the x and y components
xc1 = ((dx + abs (dx)) / 2) + 1;
xc2 = (widthx + (dx – abs (dx)) / 2);
yc1 = ((dy + abs (dy)) / 2) + 1;
yc2 = (widthy + (dy – abs (dy)) / 2);

xp1 = -((dx – abs (dx)) / 2) + 1;
xp2 = (widthx - (dx + abs (dx)) / 2);
yp1 = -((dy – abs (dy)) / 2) + 1;
yp2 = (widthy - (dy + abs (dy)) / 2);

% we subtract the z component to the DEM to get the shadow volume
temp (xp1 : xp2, yp1 : yp2) = a (xc1 : xc2, yc1 : yc2) - dz;

% at every translation we store in the final matrix the maximum of the shadow volume that was previously calculated
final = max (final, temp);
index = index + 1;

% h shows the incremental steps as in Fig 3c
set (h, 'cdata', -final / (max (max (final))));
drawnow;
end

% get and display the final results, i.e. the map showing just the shadows (as in Fig 3d)
final = final - a;
final = round((final / (max (max (final)))) + 0.499);
set(h, 'cdata', -final / (max (max (final))));
drawnow;
truesize;

8. References


